The Quantum View of the World

Nonlocality and the Kochen-Specker Paradox

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Realist construals of quantum mechanics (QM) in which all observables possess sharp values at all times, have met with two major problems. The first is posed by the Kochen-Specker paradox ([1]) in which value assignments to appropriate observables subject to a constraint known as FUNC (Functional Composition Principle) lead to an algebraic contradiction. The second problem arises from the work of Bell ([2]) which shows that in the case of two spatially separated systems the correct QM correlations between the two systems can in general only be obtained by violating locality, i.e. assuming that the value of an observable pertaining to one system depends on what sort of measurement procedure is performed on the other system. In this paper we show how these results are linked by giving a demonstration of nonlocality which does not involve consideration of correlation functions as in Bell's work, but consists in demonstrating a Kochen-Specker type of paradox on one of two separated systems under the assumption of certain locality principles. Specifically what we show is the following:

$$\text{FUNC}^* \;\&\; \text{VR} \;\&\; \text{OLOC} \;\&\; \text{ELOC} \Longrightarrow \text{Contradiction}$$

where the four principles involved in deriving the contradiction will now be explained.

We use the following notation. For any observable Q we denote the value of Q in the QM state ϕ by $[Q]^{\phi}$. Suppose B and C are two maximal observables and A is a third observable such that the associated self-adjoint operators $\hat{B}$, $\hat{C}$ and $\hat{A}$ satisfy the functional relationships $\hat{A} = f(\hat{B}) = g(\hat{C})$. Then we distinguish observables A_B, A_C where $[A_B]^{\phi} \underset{Df}{=} f[B]^{\phi}$ and $[A_C]^{\phi} \underset{Df}{=} g([C]^{\phi})$. The assumption that when $\hat{B}$ and $\hat{C}$ do not commute then the values $[A_B]^{\phi}$ and $[A_C]^{\phi}$ are in general distinct enables one to resolve the Kochen-Specker Paradox without giving up realism ([3]). We can now formulate

FUNC* : $[A_B]^{\phi} = [A_C]^{\phi}$ if B and C are maximal commuting observables.

This is a specialization of FUNC which says that the above equality holds even when B and C do not commute.

We now turn to the so-called Value Rule VR

$$\underline{\text{VR}} \quad : \quad P_Q^{\phi}(\lambda) = 0 \Longrightarrow [Q]^{\phi} \neq \lambda$$

where $P_Q^{\phi}(\lambda)$ denotes the QM probability of finding the value λ on measuring the observable Q in the state ϕ.

In order to understand OLOC and ELOC we first introduce two senses of contextuality for observables:

Ontological Contextuality arises from distinguishing the quantities A_B and A_C in the way described above.

Environmental Contextuality recognizes that the values of these quantities may depend on the environment, in particular on the apparatus set to measure some maximal observable on the system.

We apply these notions to two spatially separated systems and arrive at two quite distinct locality principles.

> OLOC: Locally maximal physical magnitudes on either of two spatially separated systems are not 'split' by ontological contextuality relative to the specification of different maximal physical magnitudes for the joint system.
>
> ELOC: The value possessed by a local physical magnitude cannot be changed by altering the arrangement of a remote piece of apparatus which forms part of the measurement context for the joint system.

Note that in terms of measurement results OLOC and ELOC cannot be distinguished. The violation of either demonstrates a dependence of the outcome recorded by an apparatus connected to one system on the setting of the apparatus connected to the other (remote) system.

OLOC is not assumed in specifying ELOC, but ELOC is only properly a locality principle if OLOC obtains. This is because violation of OLOC means we cannot specify a locally maximal magnitude independently of properties relating to the whole combined system. This leads to an ontological holism in which it is impossible to make sense of a realist construal of QM that associates properties independently with each of two separated systems.

If OLOC is assumed then violation of ELOC is the sort of nonlocality envisaged in the Bell-type nonlocality arguments.

Assuming VR and the innocent FUNC* we are forced then to

some form of nonlocality expressed in the violation of ELOC and/or OLOC. This nonlocality proof makes minimal and transparent use of probability theory (via VR) and in this respect is superior to the Bell proof. However the simplest system to which the proof applies is two spin-1 systems as compared with the two spin-$\frac{1}{2}$ systems of the Bell proof. This is because the contradiction demonstrated in our proof is closely related to, although not identical with, the Kochen-Specker paradox, for one of the two separated systems, and this is well-known to occur only if the dimension of the associated Hilbert space is greater than two.

References

[1] Kochen, S and Specker, E.P. "The Problem of Hidden Variables in Quantum Mechanics", Journal of Mathematics and Mechanics 17 (1967) 59-67.

[2] Bell, J.S. "On the Einstein-Podolsky-Rosen Paradox" Physics 1 (1964) 195-200.

[3] Van Fraassen, B. "Semantic Analysis of Quantum Logic" in C.A. Hooker (ed) Contemporary Research in the Foundations and Philosophy of Quantum Theory. D. Reidel, Dordrecht, (1973) 80-113.